Double electron capture in ^{156}Dy, ^{162}Er and ^{168}Yb

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Abstract

The double electron capture half-lives of ^{156}Dy, ^{162}Er and ^{168}Yb are evaluated using the pseudo SU(3) model, which describes ground and excited bands as well as their B(E2) and B(M1) transition strengths in remarkable agreement with experiment. The best candidate for experimental detection is the decay $^{156}\mathrm{Dy} \rightarrow {}^{156}\mathrm{Gd}$, with $\tau^{1/2}(0^+_{gs} \rightarrow 0^+_{gs}) = 2.74 \times 10^{22}$ yrs and $\tau^{1/2}(0^+_{gs} \rightarrow 0^+_1) = 8.31 \times 10^{24}$ yrs.

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The double beta decay and its relationship with weak-interactions and neutrino physics have been intensively studied in recent years. Measuring $\beta^-\beta^-{}_{2\nu}$ lifetimes of the order $10^{19} - 10^{22}$ yrs and establishing the limit $\langle m_{\nu_e} \rangle \leq 1$eV for the Majorana mass of the neutrino are major experimental achievements [1].

The double positron emitting decay ($\beta^+\beta^+{}_{2\nu}$) and the accompanying electron capture (EC) processes have been more elusive. Many candidates have low natural abundances or Q-values [2]. While the relevance of $\beta^+\beta^+{}_{0\nu}$ as a lepton violating process has been known for many years [3], only recently nuclear matrix elements were calculated in the context of the pn-QRPA [4]. Searches for these decays, both with and without neutrino emission, have been performed in ^{54}Fe, ^{78}Kr, ^{92}Mo, ^{106}Cd and ^{130,132}Ba [5].

Double electron capture processes (ECEC$_{2\nu}$) have larger Q-values, but the decay to the ground state of the final nuclei ($0^+_{gs} \rightarrow 0^+_{gs}$) has only X-rays emitted, making its detection difficult [1]. The ECEC$_{2\nu}$ decay to an excited state in the final nuclei ($0^+_{gs} \rightarrow 0^+_1$) could be detected through the two characteristic gamma rays emitted by the final nuclei [6]

In the present letter we study the ECEC$_{2\nu}$ of ^{156}Dy, ^{162}Er and ^{168}Yb using the pseudo SU(3) model. The motivation for this study is twofold: i) The first two nuclei have been mentioned as possible candidates for experimental detection, with half-lives calculated using rough estimates for the nuclear matrix elements [6]. ii) The QRPA has undesirable features which makes its predictions unreliable [7], while shell model calculations provide more reliable nuclear matrix elements for light and medium mass nuclei [7, 8]. The pseudo SU(3) shell model describes many features of heavy deformed nuclei, as ground and excited bands, B(E2) and B(M1) transition strengths in remarkable agreement with experiment using a symmetry truncated shell model theory with Hamiltonians which includes single particle energies, quadrupole-quadrupole and pairing interactions. The same formalism has proved its effectiveness for even-even [9] and odd-even [10] nuclei.

As mentioned above the $\beta^+\beta^+{}_{2\nu}$ decay can occur in three different ways:

1. The $\beta^+\beta^+{}_{2\nu}$ proper
$$(A, Z+2) \longrightarrow (A, Z) + 2e^+ + 2\nu \tag{1}$$
is easy to detect through the annihilation of the two positrons, but strongly suppressed by the low Q-value and the Coulomb repulsion between the positrons and the atomic nuclei.

2. The $\beta^+ EC_{\,2\nu}$, which captures one bound electron e_b^-
$$e_b^- + (A, Z+2) \longrightarrow (A, Z) + e^+ + 2\nu \tag{2}$$
shares with first one the hindering factors.

3. The ECEC$_{2\nu}$ double electron capture
$$2e_b^- + (A, Z+2) \longrightarrow (A, Z) + 2\nu \tag{3}$$
has the largest Q-values, no Coulomb suppression but is very difficult to detect, because only two X rays are emitted together with the neutrinos.

The double electron capture decay to excited states in the final nuclei
$$(A, Z+2) + 2e_b^- \longrightarrow \quad (A, Z)^* + 2\nu \tag{4}$$
$$\hookrightarrow (A, Z) + 2\gamma \tag{5}$$
has been proposed as a good candidate to be measured [3, 6]. The two gammas are far easier to detect than the X rays. A sensitivity close to $\sim 10^{22}$yr has been estimated for this type of experiments [6].

The decay scheme for ^{156}Dy is shown in Fig. 1. It can proceed directly to the ground state of ^{156}Gd, or to two excited 0^+ states, which decay mainly to the first 2^+ state emitting

960.55 keV or 1079.24 keV gammas, followed by a gamma with energy 88.97 keV from the decay of the 2^+ state [11]. The two gammas in coincidence are the signature of the double electron capture process.

In the pseudo $SU(3)$ shell model coupling scheme [12] normal parity orbitals (η, l, j) are identified with orbitals of a harmonic oscillator of one quanta less $\tilde{\eta} = \eta - 1$. This set of orbitals with $\tilde{j} = j = \tilde{l} + \tilde{s}$, pseudo spin $\tilde{s} = 1/2$ and pseudo orbital angular momentum $\tilde{l}$ define the so called pseudo space. The orbitals with $j = \tilde{l} \pm 1/2$ are nearly degenerate. For configurations of identical particles occupying a single j orbital of abnormal parity a convenient characterization of states is made by means of the seniority coupling scheme.

The many particle states of n_α nucleons in a given shell η_α, $\alpha = \nu$ or π, can be defined by the totally antisymmetric irreducible representations $\{1^{n_\alpha^N}\}$ and $\{1^{n_\alpha^A}\}$ of unitary groups. The dimensions of the normal (N) parity space is $\Omega_\alpha^N = (\tilde{\eta}_\alpha + 1)(\tilde{\eta}_\alpha + 2)$ and that of the unique (A) space is $\Omega_\alpha^A = 2\eta_\alpha + 4$ with the constraint $n_\alpha = n_\alpha^A + n_\alpha^N$. Proton and neutron states are coupled to angular momentum J^N and J^A in both the normal and unique parity sectors, respectively. The wave function of the many-particle state with angular momentum J and projection M is expressed as a direct product of the normal and unique parity ones, as:

$$|JM >= \sum_{J^N J^A} [|J^N > \otimes |J^A >]^J_M \tag{6}$$

We are interested in describing the low-lying energy states. In the normal subspace only pseudo spin zero configurations are taken into account. Additionally in the abnormal parity space only seniority zero configurations are taken into account. This simplification implies that $J_\pi^A = J_\nu^A = 0$. This is a very strong assumption quite useful in order to simplify the calculations. It is possible to describe the intruder states using an SU(3) formalism in the *real* space including S = 0 and 1 states [13]. Calculations performed in this scheme involve a more sophisticated formalism which we are just starting to explore.

The double beta decay, when described in the pseudo SU(3) scheme, is strongly dependent on the occupation numbers for protons and neutrons in the normal and abnormal parity states $n_\pi^N, n_\nu^N, n_\pi^A, n_\nu^A$ [14]. These numbers are determined filling the Nilsson levels from below, as discussed in [14]. In particular the $\beta^+\beta^+$ decay is allowed only if they fulfil the following relationships

$$\begin{array}{ll} n^A_{\pi,f} = n^A_{\pi,i} - 2 \;\; , & \qquad n^A_{\nu,f} = n^A_{\nu,i} \;\; , \\ n^N_{\pi,f} = n^N_{\pi,i} \;\; , & \qquad n^N_{\nu,f} = n^N_{\nu,i} + 2 \;\; . \end{array} \tag{7}$$

If these relations are not satisfied the $\beta^+\beta^+$ decay becomes forbidden. This is the first selection rule the pseudo SU(3) formalism imposes on the double beta decay, similar to that found for $\beta^-\beta^-$ processes [14].

In ^{156}Gd there is one dominant component in the ground state wave function[9]. Assuming a small deformation to satisfy Eq. (7) the ground state of this nuclei can be described as

$$
\begin{aligned}
|{}^{156}\mathrm{Gd}, 0^+\rangle \approx \;& |\;\; \{2^5\}_\pi\,(10,4)_\pi,\ \{2^4\}_\nu\,(18,4)_\nu;\ (28,8)1\,L = M = 0 >_N \\
& |\;\; (h_{11/2})^6_\pi\, J^A_\pi = 0,\ (i_{13/2})^4_\nu\, J^A_\nu = 0 >_A .
\end{aligned}
\tag{8}
$$

In a similar way, the ground state of ^{156}Dy will be dominated by

$$
\begin{aligned}
|{}^{156}\mathrm{Dy}, 0^+\rangle \approx \;& |\;\; \{2^5\}_\pi\,(10,4)_\pi;\ \{2^3\}_\nu\,(18,0)_\nu;\ (28,4)1\,L = M = 0 >_N \\
& |\;\; (h_{11/2})^6_\pi\, J^A_\pi = 0,\ (i_{13/2})^2_\nu\, J^A_\nu = 0 >_A .
\end{aligned}
\tag{9}
$$

The inverse half life of the two neutrino mode of the $ECEC_{2\nu}$-decay can be expressed in the form [15]

$$
\left[\tau^{1/2}_{2\nu}(0^+ \rightarrow 0^+_\sigma)\right]^{-1} = G_{2\nu}(\sigma) \mid M_{2\nu}(\sigma) \mid^2 \quad . \tag{10}
$$

where σ= g.s., 1 or 2 labels the final state 0^+_σ, $G_{2\nu}(\sigma)$ are kinematical factors which depend on $E_\sigma = \frac{1}{2}[Q_{\beta\beta} - E(0^+_\sigma)]$ which is half of the total energy released when the electron binding energies are neglected. The nuclear matrix element is evaluated using the pseudo SU(3) formalism [16]. For the $^{156}\mathrm{Gd} \rightarrow {}^{156}\mathrm{Dy}$ case it can be written as

$$
\begin{aligned}
M^{GT}_{2\nu}(\sigma) \;=\;& a\; b(n^A_\pi)\; \mathcal{E}^{-1}_\sigma \\
& \sum_{(\lambda_0\mu_0)K_0} < (\tilde\eta 0)1\tilde l, (\tilde\eta 0)1\tilde l \| (\lambda_0\mu_0)00 >_1 \sum_\rho < (18,0)1\; 0, (\lambda_0\mu_0)K_0 J \| (18,4)1J >_\rho \\
& \sum_{\rho'} \begin{bmatrix} (10,4) & (0,0) & (10,4) & 1 \\ (18,0) & (\lambda_0\mu_0) & (18,4) & \rho' \\ (28,4) & (\lambda_0\mu_0) & (\lambda\mu)_\sigma & \rho \\ 1 & 1 & 1 & \end{bmatrix} < (18,4) \;|||\; [a^\dagger_{\tilde\eta 0),\frac{1}{2}} a^\dagger_{\tilde\eta 0),\frac{1}{2}}]^{(\lambda_0\mu_0)} \;|||\; (18,0) >_{\rho'}
\end{aligned}
\tag{11}
$$

In the above formula $< .., .. \|, , >$ denotes the SU(3) Clebsch-Gordan coefficients [17], the symbol [...] represents $9-\lambda\mu$ recoupling coefficients [18], $< ..\; |||\; ..\; |||\; .. >$ are the triple reduced matrix elements [19] and the following notation was introduced:

$$
\begin{aligned}
& a = \frac{4\eta}{(2\eta+1)\sqrt{2\eta-1}}, \qquad\qquad b(n^A_\pi) = [n^A_\pi(\eta + 2 - n^A_\pi/2)]^{1/2}, \\
& \mathcal{E}_\sigma = E_\sigma - \hbar\omega k_\pi 2 j_\pi + \Delta_C \quad \text{with} \quad \Delta_C = \frac{0.70}{A^{1/3}}[2Z + 1 - 0.76((Z+1)^{4/3} - Z^{4/3})]
\end{aligned}
\tag{12}
$$

The SU(3) tensorial components (λ_0, μ_0) of the normal part of the double Gamow-Teller operator must be able to couple the proton and total irreps (18,0) and (28,4) associated with

the ground state of ^{156}Gd to the corresponding irreps (18,4) and $(\lambda\mu)_\sigma$ = (28,8), (30,4) and (32,0), which characterize the ground and excited rotational bands in ^{156}Gd. If these irreps cannot be connected by (λ_0, μ_0) the $\beta^+\beta^+$ decays to the 0^+ states are forbidden. This is a second selection rule imposed by the model to the $\beta\beta$ decay.

Transition		$G_{2\nu}(\mathrm{yr}^{-1})$	$\mid M_{2\nu}(\sigma) \mid$	$\tau^{1/2}(\mathrm{yr})$
^{156}Dy $\rightarrow ^{156}$Gd	$0^+ \rightarrow 0^+(g.s.)$	9.79×10^{-21}	0.061	2.74×10^{22}
	$0^+ \rightarrow 0^+(1)$	1.65×10^{-22}	0.027	8.31×10^{24}
	$0^+ \rightarrow 0^+(2)$	7.58×10^{-23}	0.035	1.08×10^{25}
^{162}Er $\rightarrow ^{162}$Dy	$0^+ \rightarrow 0^+(g.s.)$	8.06×10^{-21}	0.066	2.85×10^{22}
	$0^+ \rightarrow 0^+(1)$	1.60×10^{-24}	0.013	3.70×10^{27}
^{168}Yb $\rightarrow ^{168}$Er	$0^+ \rightarrow 0^+(g.s.)$	2.47×10^{-21}	0.045	2.00×10^{23}
	$0^+ \rightarrow 0^+(1)$	5.18×10^{-28}	0.0006	5.36×10^{33}

Table 1: Half-lives for the $ECEC_{2\nu}$ decay to the ground and excited states of the final nuclei.

In Table 1 we present the $ECEC_{2\nu}$ decay of ^{156}Dy, ^{162}Er and ^{168}Yb to the ground and excited states of ^{156}Gd, ^{162}Dy and ^{168}Er respectively. The kinematical factors $G_{2\nu}(\sigma)$ were evaluated following the prescriptions given in [20]. When the energy released in the decay to an excited stated $(2E_\sigma)$ is small the available phase space $G_{2\nu}$ is strongly reduced, and the half life could be very large. It is the case in the double electron capture decay to the first excited 0^+ state in ^{168}Er. Details of the calculation of the nuclear matrix elements, as well as the energy spectra of the nuclei involved will be presented elsewhere [21]. The nuclear matrix elements associated with the decay to the ground state of the final nuclei $|M_{2\nu}(g.s.)|$ have values close to 0.05 - 0.06, a factor of 5 smaller than the assumption of Barabash [6], and are similar for the three nuclei studied. The nuclear matrix elements to excited states $|M_{2\nu}(1,2)|$ show a wide spread, being close to those of the ground state for ^{156}Dy, suppressed by a factor 5 for ^{162}Er and by a factor 80 for ^{168}Yb. While in general it is confirmed that deformed nuclei have smaller nuclear matrix elements than spherical [14], ^{156}Dy appears to be the best candidate of this group for experimental detection, with a half-life around 10^{24} years for the double electron capture to the first excited 0^+ state.

In summary, we have used the pseudo SU(3) shell model to investigate the double electron capture decays in three heavy deformed nuclei, and found that experiments with sensitivities around 10^{24} years could detect the decay of ^{156}Dy to the first excited state in ^{156}Gd, while detecting the doubled electron capture decay in ^{162}Er and ^{168}Yb would be very difficult.

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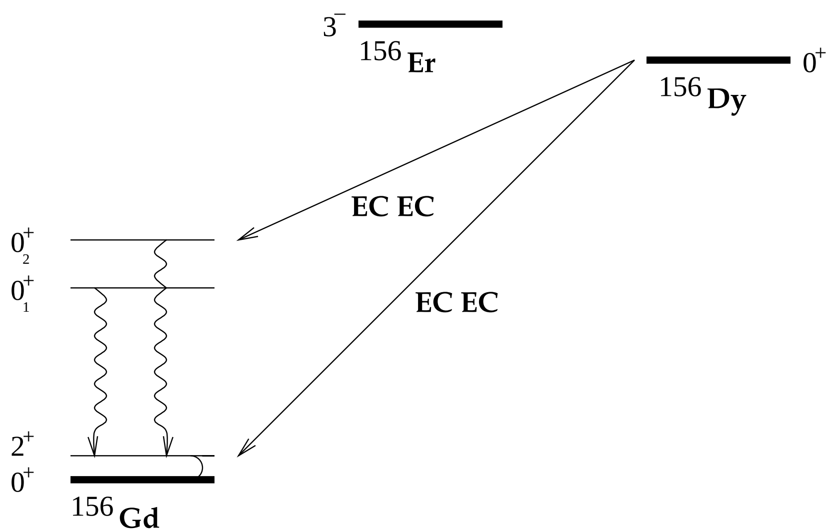
Figure Captions:
Energy levels relevant for the $\beta^+\beta^+$ decay of ^{156}Dy.



3^-
^{156}Er
0^+
^{156}Dy
EC EC
EC EC
0^+_2
0^+_1
2^+
0^+
^{156}Gd